Name: _____

- This packet is due at **12:00 pm PDT (noon) on Friday, May 14**.
- Neatly fill out this packet either directly or on your own paper. Then upload your answers to Canvas. Please clearly number each problem and keep them in order.
- For each problem in this packet, **organize and show all of your work and justification** for your answer.
- Make sure that it is clear what your final answer is. This may mean boxing/circling it or it may mean doing neither.
- If you are required to draw a graph of a function, each axis must be labeled and given a scale.
- I will not be grading every individual problem, but answers will be available on Canvas after the due date.
- I am happy to help you however I can. Here are some options:
  - Come to my office hours (see syllabus).
  - Email me directly at gknapp4@uoregon.edu

1. Find the radius and interval of convergence for the following power series.

(a) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n}$

R. T. $\lim_{n\to\infty} |a_n|^{1/n} = |2x|$

↳ Want: $|2x| < 1$ (b/c this gives conv.)

$|x| < \frac{1}{2}$

x within $\frac{1}{2}$ of 0

$x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ⟶ Conv.

A.S.T: alt.

$|b_n| \to 0$ monotonically (dec.)

(b) $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{\sqrt{n}}$

p-test: $\sum \frac{1}{n^p}$

$|-x| < 1$

$R = 1$

$[(-1)^n]^2 = (-1)^{2n}$

$= [(-1)^2]^n$

$= 1^n = 1$

$$\sum_{n=1}^{\infty} \frac{[(-1)^n]^2}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$$\sum (-1)^n \cdot \frac{1}{\sqrt{n}}$$

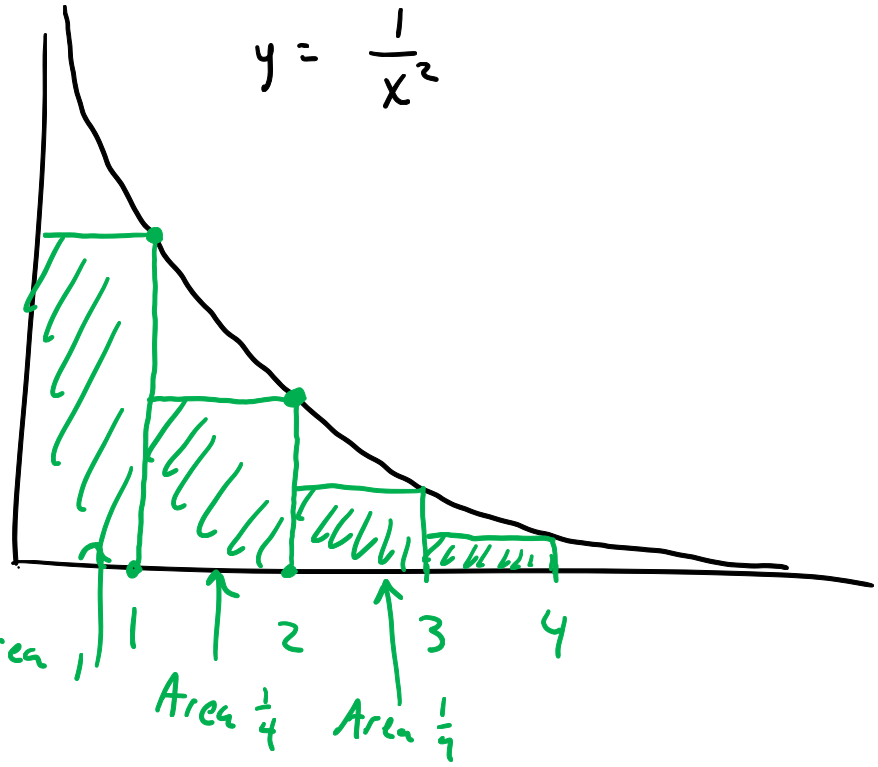
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{\sqrt{n}} = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots$$

$(-1)^n \cdot (-1)^n = (-1)^{2n} = 1$ ← even

$$\sum_{n=} \frac{1}{n}$$

$I = (-1, 1]$

(c) $\sum_{n=1}^{\infty} \frac{10^n x^n}{n!}$



$\sum \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

↑ area of boxes

$\int_1^\infty \frac{1}{x^2}\,dx$ = finite #

- $\lim \dots = 0$ Convergence when $\lim < 1$

$I = (-\infty, \infty)$

$R = \infty$

- $\lim \dots = \infty$

$I = [a, a]$

$R = 0$

(d) $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{\ln(2n)}$

$$\lim_{n\to\infty}\left|-x\cdot\frac{\ln(2n)}{\ln(2n+2)}\right|$$

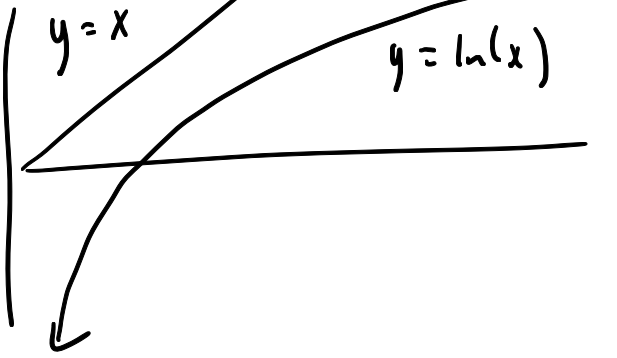
$$\lim_{n\to\infty}\frac{\ln(2n)}{\ln(2n+2)} \neq \ln\ln\left(\frac{2n}{2n+2}\right)$$

$$\lim_{n\to\infty}\frac{\left(\frac{2}{2n}\right)}{\left(\frac{2}{2n+2}\right)}$$

$$\ln\left(\frac{a}{b}\right)=\ln(a)-\ln(b)$$

$$\frac{d}{dx}\left(\ln(x)\right)=\frac{1}{x}$$

$$\sum\frac{1}{\ln(2n)}$$



$$\ln(x)\leq x$$

$$\frac{1}{\ln(x)}\geq\frac{1}{x}$$

Fact: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n$

2. Find the **radius** of convergence for each of the following series. You do not need to find the actual interval.

(a) $\sum_{n=1}^{\infty} \frac{n!(x+5)^n}{n^n}$

$\frac{a^b}{a^c} = a^{b-c}$

$\frac{b^a}{c^a} = \left(\frac{b}{c}\right)^a$

$$\lim_{n\to\infty}\left|\frac{(n+1)(x+5)\cdot n^n}{(n+1)^{n+1}}\right|$$

$$= \lim_{n\to\infty}\left|(x+5)\frac{n^n}{(n+1)^n}\right|$$

$$= \lim_{n\to\infty}\left|(x+5)\left(\frac{n}{n+1}\right)^n\right|$$

$$= \lim_{n\to\infty}\left|(x+5)\frac{1}{\left(\frac{n+1}{n}\right)^n}\right|$$

$$= \left|\frac{x+5}{e}\right| < 1$$

$$|x+5| < e$$

$\frac{d}{dx}\left(e^x\right) = e^x$

Fact: $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

(b) $\sum_{n=1}^{\infty} \frac{(n!)^3(2x-4)^n}{(3n)!}$

